Appendix for "Intervention, War Expansion, and the International Sources of Civil War"

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1 Civil war

Proposition 1 *When* $\pi < \min\{\pi^{IW}, \pi_T^{\dagger}\}$, $e > e_{CW}$, $\overline{c}_D > \overline{c}_D^{CW}$, $c_R < c^{\dagger}$, $a \leq a^{\dagger}$, $p_R^{IW} > p_R^{IW\dagger}$, *and* $b < b^{\dagger}$ *there exists a Perfect Bayesian Equilibrium in which:*

- *R challenges.*
- *D rejects and retaliates when* $c_D < c_D^{\ddagger}$, *rejects and tolerates when* $c_D^{\ddagger} \leq c_D < c_D^{\dagger}$ *and accepts when* $c_D \geq c_D^{\dagger}$.
- *If D rejects, T believes* $c_D \sim U(0, c_D^{\dagger}]$ *and does not offer support to R; otherwise T believes* $c_D \sim U(c_D^{\dagger}, \overline{c}_D]$.
- *If D rejects and T offers support, R believes* $c_D \sim U(0, c_D^{\dagger}]$ *and accepts; otherwise R believes* $c_D \sim U(c_D^{\dagger}, \overline{c}_D]$.

Proof of Proposition 1. I start by identifying D's cutpoint strategy. D's type is drawn from a uniform, continuous distribution, defined by $c_D^* \in (0, \overline{c}_D)$, and it is indifferent between acquiescing and fighting a civil war when $0 = p_D^{CW}\pi - c_D$, or $c_D^\dagger = p_D^{CW}\pi$. D is indifferent between tolerating and retaliating when $p_D^{ICW}\pi - c_D = p_D^{IW} - (ec_D)$, or $c_D^\ddagger = \frac{p_D^{IW} - p_D^{ICW}\pi}{e-1}$. D is a plausible type that retaliates after rejecting R's demand when $0 < c_D^\ddagger < c_D^\dagger < \overline{c}_D$, which is true when:

$$\pi < \frac{p_D^{IW}}{p_D^{ICW}} = \pi^{IW}, \tag{1}$$

$$e > \frac{p_D^{IW} + \pi * (p_D^{CW} - p_D^{ICW})}{p_D^{CW}\pi} = e_{CW}, \tag{2}$$

and

$$\overline{c}_D > p_D^{CW}\pi = \overline{c}_D^{CW} \tag{3}$$

Next, R believes $c_D \sim U(0, \overline{c}_D]$ and challenges when $\frac{c_D^\dagger}{\overline{c}_D}((1 - p_D^{CW})\pi - c_R) + \frac{\overline{c}_D - c_D^\dagger}{\overline{c}_D}\pi \geq 0$, which simplifies to $c_R \leq \frac{\overline{c}_D\pi}{p_D^{CW}\pi} - p_D^{CW}\pi = c_R^\dagger$. $c_R^\dagger > 0$ is ensured when $\overline{c}_D > p_D^{CW}(p_D^{CW}\pi)$, which is true as long as Equation 3 holds. I assume that war is costly for R, so $(1 - p_D^{CW})\pi - c_R < 0$, or $c_R > (1 - p_D^{CW})\pi$. There is a range of c_R where R challenges without being undeterrrable when $0 < 1 - p_D^{CW}\pi < c_R^\dagger < \frac{\overline{c}_D\pi}{p_D^{CW}\pi} - p_D^{CW}\pi$. This is always true as long as $\overline{c}_D > \overline{c}_D^{CW}$, which holds as long as Equation 3 holds.

Following D's rejection, T stays out of a civil war when $(1-\pi) \geq \frac{c_D^\ddagger}{c_D^\dagger}((1 - p_D^{IW} - p_{R_I W}) - (ec_T)) + \frac{c_D^\dagger - c_D^\ddagger}{c_D^\dagger} * ((1-\pi) + (1 - p_D^{ICW}) * b - c_T)$, which simplifies to $b < -\frac{c_T(c_D^\dagger + c_D^\ddagger(e-1)) + c_D^\ddagger(p_D^{IW} + p_R^{IW} - \pi)}{(p_D^{ICW} - 1)\pi(c_D^\dagger - c_D^\ddagger)} = b^\dagger$. $b^\dagger > 0$ is ensured when $\pi < p_D^{IW} + p_R^{IW} = \pi_T^\dagger$.

R accepts support if offered when it prefers fighting an international conflict to fighting alone, or $\frac{c_D^{\ddagger}}{c_D^{\dagger}}(p_R^{IW} - (ec_R)) + \frac{c_D^{\dagger} - c_D^{\ddagger}}{c_D^{\dagger}}((1 - p_D^{ICW})(\pi - a) - c_R) \geq (1 - p_D^{CW})\pi - c_R$, which simplifies to

$$a \leq \frac{c_D^{\dagger}(p_D^{ICW} - p_D^{CW})\pi - c_D^{\ddagger}((p_D^{ICW} - 1)\pi + p_R^{IW}) + c_D^{\ddagger}(e-1)c_R}{(c_D^{\dagger} - c_D^{\ddagger})(p_D^{ICW} - 1)} = a^{\dagger}. \tag{4}$$

$a^{\dagger} > 0$ is ensured when $p_R^{IW} > \pi - p_D^{ICW}\pi + \frac{c_D^{\dagger}(p_D^{ICW} - p_D^{CW})\pi}{c_D^{\ddagger}} + (e-1)c_R = p_R^{IW\dagger}$.

Next, I identify a PBE where R challenges D, but rejects support from T if fighting starts.

Proposition 2 *When $\pi < \min\{\pi^{IW}, \pi_T^{\dagger}\}$, $e > e_{CW}$, $\overline{c}_D > \overline{c}_D^{CW}$, $c_R < c^{\dagger}$, $a > a^{\dagger}$, $p_R^{IW} > p_R^{IW\dagger}$, and $b \geq b^{\dagger}$ there exists a Perfect Bayesian Equilibrium in which:*

- *R challenges.*
- *D rejects and retaliates when $c_D < c_D^{\ddagger}$, rejects and tolerates when $c_D^{\ddagger} \leq c_D < c_D^{\dagger}$ and accepts when $c_D \geq c_D^{\dagger}$.*
- *If D rejects, T believes $c_D \sim U(0, c_D^{\dagger}]$ and offers support to R; otherwise T believes $c_D \sim U(c_D^{\dagger}, \overline{c}_D]$.*
- *If D rejects and T offers support, R believes $c_D \sim U(0, c_D^{\dagger}]$ and rejects and fights alone; otherwise R believes $c_D \sim U(c_D^{\dagger}, \overline{c}_D]$.*

Proof of Proposition 2. D's cutpoint strategy is identical to that of Proposition 1, since T will not intervene and R's choice does not affect D's utility functions. D is indifferent between acquiescing and fighting a civil war when $c_D^{\dagger} = p_D^{CW}\pi$, and D is indifferent between tolerating and retaliating when $c_D^{\ddagger} = \frac{p_D^{IW} - p_D^{ICW}\pi}{e-1}$. D is a plausible type that retaliates after rejecting R's demand when $0 < c_D^{\ddagger} < c_D^{\dagger} < \overline{c}_D$, which holds when $\pi < \pi^{IW}$, $e > e_{CW}$, and $\overline{c}_D > \overline{c}_D^{CW}$.

Therefore, R's beliefs about D's type are also identical to those in Proposition 1. Furthermore, if T will offer help but R will decline, R challenges when $c_R \leq c_R^{\dagger}$, which holds under the same logic as in Proposition 1.

If D rejects a challenge, R and T share the same beliefs about D's type, identical to T's updated beliefs described in Proposition 1. Given these beliefs, T prefers intervention to staying out when $(1-\pi) < \frac{c_D^{\ddagger}}{c_D^{\dagger}}((1-p_D^{IW}-p_{R_IW})-(ec_T)) + \frac{c_D^{\dagger}-c_D^{\ddagger}}{c_D^{\dagger}} * ((1-\pi)+(1-p_D^{ICW}) * b - c_T)$, which simplifies to $b \geq -\frac{c_T(c_D^{\dagger}+c_D^{\ddagger}(e-1))+c_D^{\ddagger}(p_D^{IW}+p_R^{IW}-\pi)}{(p_D^{ICW}-1)\pi(c_D^{\dagger}-c_D^{\ddagger})} = b^{\dagger}$. $b^{\dagger} > 0$ is ensured when $\pi < p_D^{IW} + p_R^{IW} = \pi_T^{\dagger}$.

Following a rejection by D and an offer of support from T, R rejects external support and fights alone when it prefers fighting a civil war to fighting an international conflict, or $a > a^{\dagger}$, and $a^{\dagger} > 0$ is ensured when $p_R^{IW} > p_R^{IW\dagger}$.

Lastly, there is a PBE that also results in local-only civil war. R challenges D, and after D rejects, T does not offer support and R does not accept any support. Beliefs about D's type are identical to beliefs in Propositions 1 and 2, and the PBE is characterized by $b > b^{\dagger}$ and $a > a^{\dagger}$.

2 Internationalized civil war and interstate war

Proposition 3 *When $\pi < \pi^{IW}$, $e > \max\{e_{IW}, e_T^{\circ}, e_R^{\circ}\}$, $\overline{c}_D > \max\{\overline{c}_D^{IW}, \overline{c}_D^{\circ}\}$, $c_R^{\circ} > c_R$, $a \leq a^{\circ}$, $b \geq b^{\circ}$, and $c_T < c_T^{\circ}$, there exists a Perfect Bayesian Equilibrium in which:*

- *R challenges.*
- *D rejects and retaliates when $c_D < c_D^{\ddagger}$, rejects and tolerates when $c_D^{\ddagger} \leq c_D < c_D^{\circ}$ and accepts when $c_D \geq c_D^{\circ}$.*
- *If D fights, T believes $c_D \sim U(0, c_D^{\circ}]$ and offers support to R; otherwise T believes $c_D \sim U(c_D^{\circ}, \overline{c}_D]$.*

- *If D rejects and T offers support, R believes* $c_D \sim U(0, c_D^\circ]$ *and accepts; otherwise R believes* $c_D \sim U(c_D^\circ, \overline{c}_D]$.

Proof of Proposition 3. I start by defining D's cutpoint strategy. D is indifferent between tolerating intervention and retaliating at cut-point $c_D^\ddagger$, and is indifferent between tolerating intervention and acquiescing to R's challenge when $p_D^{ICW}\pi - c_D = 0$, which simplifies to $c_D^\circ = p_D^{ICW}\pi$. Incentive compatibility requires the cutpoints be ordered $0 < c_D^\ddagger < c_D^\circ < \overline{c}_D$, which is true when $\pi < \pi^{IW}$ (as above), $e > \frac{p_D^{IW}}{p_D^{ICW} * \pi} = e_{IW}$, and $\overline{c}_D > p_D^{ICW}\pi = \overline{c}_D^{IW}$.

R challenges D when it prefers fighting an international conflict to accepting the status quo. Given a challenge from R, there are three potential outcomes. D fights and retaliates against T, fights and tolerates an intervention, or acquiesces. R therefore challenges when:

$$\frac{c_D^\ddagger}{\overline{c}_D}(p_R^{IW} - (ec_R)) + \frac{c_D^\circ - c_D^\ddagger}{\overline{c}_D}((1 - p_D^{ICW})(\pi - a) - c_R) + \frac{\overline{c}_D - c_D^\circ}{\overline{c}_D}\pi \geq 0, \tag{5}$$

which simplifies to $c_R < \frac{a(c_D^\circ - c_D^\ddagger)(p_D^{ICW} - 1) + (\overline{c}_D + c_D^\ddagger(p_D^{ICW} - 1) - c_D^\circ p_D^{ICW})\pi + c_D^\ddagger p_R^{IW}}{c_D^\circ + c_D^\ddagger(e-1)} = c_R^\circ$. $c_R^\circ > 0$ is ensured when $\overline{c}_D > \frac{-a(c_D^\circ - c_D^\ddagger)(p_D^{ICW} - 1) + c_D^\circ p_D^{ICW}\pi - c_D^\ddagger((p_D^{ICW} - 1)\pi + p_R^{IW}}{\pi} = \overline{c}_D^\circ$.

To ensure war does not happen because R is undeterrable, R prefers the status quo to fighting, so $(1 - p_D^{ICW})(\pi - a) - c_R < 0$ and $p_R^{IW} - (e * c_R) < 0$, or $c_R > (p_D^{ICW} - 1)(a - \pi)$ and $c_R > \frac{p_R^{IW}}{e}$, respectively. For there to be a range of types R that risk war despite war being costly, $c_R^\circ > \max\{(p_D^{ICW} - 1)(a - \pi), \frac{p_R^{IW}}{e}\}$. This is true as long as $c_R^\circ > c_R > 0$, because if

$$\frac{c_D^\ddagger}{\overline{c}_D}u_R(IW) + \frac{c_D^\circ - c_D^\ddagger}{\overline{c}_D}u_R(ICW) + \frac{\overline{c}_D - c_D^\circ}{\overline{c}_D}\pi \geq 0 \tag{6}$$

and

$$0 > \frac{c_D^{\ddagger}}{\overline{c}_D} u_R(IW) + \frac{c_D^{\circ} - c_D^{\ddagger}}{\overline{c}_D} u_R(ICW), \tag{7}$$

then $\frac{c_D^{\ddagger}}{\overline{c}_D}(p_R^{IW} - (e * c_R)) + \frac{c_D^{\circ}-c_D^{\ddagger}}{\overline{c}_D}((1 - p_D^{ICW})(\pi - a) - c_R) > \frac{c_D^{\ddagger}}{\overline{c}_D} u_R(IW) + \frac{c_D^{\circ}-c_D^{\ddagger}}{\overline{c}_D} u_R(ICW)$ is true.

Following a rejection by D, R and T share the same beliefs about D's type. T offers support when $\frac{c_D^{\ddagger}}{c_D^{\circ}}((1 - p_D^{IW} - p_R^{IW}) - (e \times c_T)) + \frac{c_D^{\circ}-c_D^{\ddagger}}{c_D^{\circ}}((1-\pi) + (1 - p_D^{ICW})b - c_T) \geq (1-\pi)$, which simplifies to

$$b \geq \frac{c_T(c_D^{\ddagger}(1-e) - c_D^{\circ}) + c_D^{\ddagger}(\pi - p_D^{IW} - p_R^{IW})}{(p_D^{ICW} - 1)\pi(c_D^{\circ} - c_D^{\ddagger})} = b^{\circ}. \tag{8}$$

$0 < b^{\circ} < 1$ is ensured when $e > 1 - \frac{p_D^{IW} - \pi + p_R^{IW}}{c_T} - \frac{c_D^{\circ}}{c_D^{\ddagger}} = e_T^{\circ}$ and

$$c_T < -\frac{c_D^{\circ}(p_D^{ICW} - 1)\pi + c_D^{\ddagger}(p_D^{IW} + p_R^{IW} - p_D^{ICW}\pi)}{c_D^{\circ} + c_D^{\ddagger}(e-1)} = c_T^{\circ}. \tag{9}$$

Lastly, R accepts T's offer of help when it prefers an international conflict to fighting a civil war alone, or $\frac{c_D^{\ddagger}}{c_D^{\circ}}(p_R^{IW} - (ec_R)) + \frac{c_D^{\circ}-c_D^{\ddagger}}{c_D^{\circ}}((1 - p_D^{ICW})(\pi - a) - c_R) \geq (1 - p_D^{CW})\pi - c_R$, which simplifies to $a \leq \frac{c_D^{\ddagger}(e-1)c_R + c_D^{\circ}\pi(p_D^{ICW} - p_D^{CW}) - c_D^{\ddagger}((p_D^{ICW}-1)\pi + p_R^{IW})}{(p_D^{ICW}-1)(c_D^{\circ} - c_D^{\ddagger})} = a^{\circ}$. $a^{\circ} > 0$ is ensured when $e > \frac{c_D^{\ddagger}c_R + c_D^{\circ}\pi(p_D^{CW} - p_D^{ICW}) + c_D^{\ddagger}((p_D^{ICW}-1)\pi + p_R^{IW})}{c_D^{\ddagger}c_R} = e_R^{\circ}$.

3 Declining support at a cost

Propositions 1 and 2 show that autonomy costs do not affect the risk of civil war, only the type of conflict that occurs in equilibrium. However, autonomy costs do shape the risk of rebellion if rejecting T's support is costly and correlated with R's autonomy costs, which is a plausible assumption. For instance, weaker groups are more likely to be dominated if they receive support, and they might also suffer some costs from turning down help, in the form of infighting or sanctions by the third party. To show that, consider a PBE with the same strategies and beliefs as Proposition 2, but R pays a cost $(a \times r)$ for declining support where $0 < r < 1$. R challenges (but prefers fighting alone to receiving support) when $\frac{c_D^{\dagger}}{\overline{c}_D}((1-p_D^{CW})\pi - c_R - (a \times r)) + \frac{\overline{c}_D - c_D^{\dagger}}{\overline{c}_D}\pi \geq 0$, which simplifies to $c_R \leq \frac{\overline{c}_D}{p_D^{CW}} - p_D^{CW}\pi - (ar) = c_R^{\oplus}$. Because $\frac{\partial c_R^{\oplus}}{\partial a} < 0$, R is less likely to challenge the higher its autonomy costs are. Furthermore, R declines support following a rejection when $((1-p_D^{CW})\pi - c_R - (a \times r)) \geq \frac{c_D^{\ddagger}}{c_D^{\dagger}}(p_R^{IW} - (ec_R)) + \frac{c_D^{\dagger} - c_D^{\ddagger}}{c_D^{\dagger}}((1-p_D^{ICW})(\pi - a) - c_R)$, which simplifies to $a > a^{\oplus}$, so higher autonomy costs make R more likely to decline support as well.

A correlation between autonomy costs and costs of declining support can explain existing empirical patterns of rebellion. For instance, secessionist groups are more likely to rebel than other groups (Cederman et al. 2010, p. 105), which could be explained by the fact that hese ethnic groups have the internal cohesion and institutions to withstand pressure from a third party. Furthermore, if autonomy costs are particularly high, intervention can deter R from rebelling in the first place, and a credible threat of retaliation can thus encourage some high autonomy-cost groups to rebel by deterring intervention. If D's ability to deter intervention is negatively correlated with R's cohesion (i.e. rebels are more likely to be factionalized in the face of a strong government), this implication can account for the empirical finding that internal division within groups increase the risk of civil war (Cunningham 2013). As groups become more divided, they are less able to withstand pressure from a third party, especially if one or more factions have ties with a third party. A stronger government thus deters intervention, but in doing so risks a challenge from rebels that would otherwise be deterred by intervention.

4 Varying levels of intervention

In this section I relax the assumption that the level of intervention is fixed. Instead, T can choose to conduct either a small or large intervention, which increases R's chances of winning in an internationalized civil war a little or a lot (proofs in the appendix). To isolate the relationship between the risk of war and varying levels of intervention, I assume T's costs of intervention stay the same regardless, as does R's autonomy costs. Furthermore, the level of intervention does not affect payoffs in interstate war, only the probability of ending up in one. Lastly, the level of intervention is granular rather than continuous to keep the analysis tractable. T therefore does not have the option to optimize the level of intervention, but seeing as how I am interested in how the key parameters affect T's choice to risk a little or a lot, two levels rather than infinite will suffice.[1]

Varying levels of intervention affect D's strategies in a straightforward manner. A large intervention reduces the probability of war relative to a small intervention relative to no intervention. Third parties who can set their support can more effectively compel domestic governments into acquiescing to rebel demands. However, a large intervention increases the likelihood of retaliation relative to a small one, because it diminishes the difference between D winning an interstate war and an internationalized civil war, thus making the added benefits of war expansion more attractive.

As for the rebels, a large intervention reduces the risk of war and increases its chances of winning an internationalized civil war, but increases the likelihood of an interstate war. Assuming that R wants support, the effect on R's willingness to challenge depends on various factors, such as its autonomy costs. When autonomy costs are low, higher levels of support make R less willing to challenge D, because it wants to avoid war expansion. Conversely, when autonomy costs are high, higher levels of support make R more willing to challenge, because it increases the likelihood of war expansion.

If T is given the option of intervening small (p_D^{ICWs}) or large (p_D^{ICWl}), where $p_D^{ICWs} > p_D^{ICWl}$, there exists at least one PBE where T provides a small amount of support and R rebels. Since R's strategy follows the same logic as Proposition 3, I focus here on T's strategy. T intervenes when it is not too costly (as above), but chooses the level of support depending on how much intervention helps R win and how this affects the risk of retaliation. D is indifferent between tolerating a small intervention and acquiescing to R's challenge when $p_D^{ICWs}\pi - c_D = 0$, which simplifies to $c_D^{\otimes} = p_D^{ICWs}\pi$, and indifferent between tolerating a small intervention and retaliating when $p_D^{ICWs}\pi - c_D = p_D^{IW} - (e \times c_D)$, or $c_D^{\star} = \frac{p_D^{IW} - p_D^{ICWs}\pi}{e-1}$. D is indifferent between tolerating a large intervention and retaliating when $p_D^{ICWl}\pi - c_D = p_D^{IW} - (ec_D)$, or $c_D^{\bullet} = \frac{p_D^{IW} - p_D^{ICWl}\pi}{e-1}$. Therefore, T provides a small level of support when:

$$\begin{aligned}
&\frac{c_D^{\star}}{c_D^{\otimes}}((1 - p_D^{IW} - p_{R_I W}) - (e \times c_T)) + \frac{c_D^{\otimes} - c_D^{\star}}{c_D^{\circ}}((1-\pi) + (1 - p_D^{ICWs})b - c_T) \\
\geq &\frac{c_D^{\bullet}}{c_D^{\otimes}}((1 - p_D^{IW} - p_{R_I W}) - (e \times c_T)) + \frac{c_D^{\otimes} - c_D^{\bullet}}{c_D^{\otimes}}((1-\pi) + (1 - p_D^{ICWl})b - c_T)
\end{aligned} \tag{10}$$

Setting the two utility functions as equal and solving for p_D^{ICWl} yields two solutions: p_D^{ICWs} and $\frac{\pi(b(-e)p_D^{ICWs} + b - 1) + (b+1)p_D^{IW} + (e-1)c_T + p_R^{IW}}{b\pi}$. Because the RHS of the inequality is a concave function of p_D^{ICWl}, two cut-points imply a range of p_D^{ICWl} where defection to a large intervention is profitable, but the first solution is a trivial constraint, because $p_D^{ICWl} < p_D^{ICWs}$ by assumption, so p_D^{ICWs} must be an upper bound for defection. The second solution is therefore a lower bound for defection, which implies that T does not defect to a larger intervention when the gap between a small and a large is too great, because doing so will surely provoke retaliation.

This implies that whether T conducts a large or small intervention, depends on the distance between the two levels. T chooses a small intervention when a larger intervention helps too much. When escalation is particularly granular, a large jump in rebel support means a large decline in D's chances of winning an internationalized civil war, which in turn increases the risk of retaliation. Past a certain point, helping R more cannot make up for the increased likelihood of interstate war. An implication of this result is that third parties should be particularly wary of introducing new technology that has a substantial effect on the probability of rebel victory.

5 Interdependence and club goods

In this section I show how the results of the model may or may not be affected by changes to R's and T's payoffs as they pertain to the generation of club goods. First, if R's willingness to challenge is a function of affinity (e.g. $f(a) = \frac{a}{b}$), then $\frac{\partial u_R(ICW)}{\partial b} > 0$, which means the likelihood of rebellion and intervention are increasing in affinity.

Second, if T's payoff in an internationalized civil war is an increasing function of R's autonomy costs, then T intervenes when:

$$\frac{c_D^{\ddagger}}{c_D^{\circ}}((1 - p_D^{IW} - p_R^{IW}) - (e \times c_T)) + \frac{c_D^{\circ} - c_D^{\ddagger}}{c_D^{\circ}}((1 - \pi) + (1 - p_D^{ICW})ba - c_T) \geq (1 - \pi), \tag{11}$$

which simplifies to: $\frac{c_T(c_D^{\ddagger}(e-1)+c_D^{\circ})+c_D^{\ddagger}(p_D^{IW}-\pi+p_R^{IW})}{b(p_D^{ICW}-1)\pi(c_D^{\ddagger}-c_D^{\circ})} > a = a_T^*$. This is then the lower bound for a in equilibrium.

If T is intervening, R challenges when:

$$\frac{c_D^{\ddagger}}{\overline{c}_D}(p_R^{IW} - (ec_R)) + \frac{c_D^{\circ} - c_D^{\ddagger}}{\overline{c}_D}((1 - p_D^{ICW})(\pi - \frac{a}{b}) - c_R) + \frac{\overline{c}_D - c_D^{\circ}}{\overline{c}_D}\pi \geq 0, \tag{12}$$

which I rearrange as $\frac{b\left(-c_R(c_D^{\ddagger}(e-1)+c_D^{\circ})+\pi(\overline{c}_D+c_D^{\ddagger}(p_D^{ICW}-1)-c_D^{\circ}p_D^{ICW})+c_D^{\ddagger}p_R^{IW}\right)}{(p_D^{ICW}-1)(c_D^{\ddagger}-c_D^{\circ})} < a = a_R^*$. R challenges and T intervenes when $a_T^* \leq a \leq a_R^*$, which holds when b and $\overline{c}_D$ are sufficiently high, and c_T and c_R are sufficiently low.

Third, to show that allowing T to receive club goods without intervening does not change the results of the model, it is sufficient to compare the comparative statics of T's payoffs for internationalized civil war and civil war in terms of b and show that the former is greater than the latter. By assumption, $\frac{\partial u_T(ICW)}{\partial b} = \pi - p_D^{ICW}\pi > \frac{\partial u_T(CW)}{\partial b} = \pi - p_D^{CW}\pi$.

6 Coalition war-fighting

Relatedly, it is trivial to show that allowing R and T to fight in a coalition after retaliation, rather than in an all-against-all interstate war, does not affect why they fight. Consider first a situation where R and T fight together against D in an interstate war, and then split up the winnings based on their relative military strength, without any efficiency gains or generation of club goods. If so, their respective payoffs for fighting an interstate war are $u_R(IW) = \frac{m_R+m_T}{m_D+m_R+m_T} \times \frac{m_R}{m_R+m_T} - (e \times c_R)$ and $u_T(IW) = \frac{m_R+m_T}{m_D+m_R+m_T} \times \frac{m_T}{m_R+m_T} - (e \times c_T)$, which simplify to $u_R(IW) = \frac{m_R}{m_D+m_R+m_T} - (e \times c_R)$ and $u_T(IW) = \frac{m_R}{m_D+m_R+m_T} - (e \times c_T)$. As such, the payoffs for fighting a coalition war without coalition benefits and fighting an all-against-all war are identical for both actors.

Next, consider a situation where R and T enjoy some club good generated after winning an interstate war. For R, $p_R^{IW} + (1 - p_D^{IW} - p_R^{IW})b - (e \times c_R) > p_R^{IW} - (e \times c_R)$ by assumption. Similarly, for T, $(1-p_D^{IW}-p_R^{IW})+p_R^{IW}b-(e\times c_T) > (1-p_D^{IW}-p_R^{IW})-(e\times c_T)$ is always true. While coalition fighting offers club goods, it does not change why they fight, because b is bound between 0 and 1, so neither of the actors prefer the other to win at the expense of themself.

Lastly, R and T might enjoy some efficiency gains ($w > 0$) from fighting together. If so, their probability of winning an interstate war is $\frac{m_R+m_T+w}{m_D+m_R+m_T+w}$. If we assume they divide up the spoils according to their relative military strengths, their respective payoffs for fighting an interstate war are $u_R(IWc) = \frac{m_R+m_T+w}{m_D+m_R+m_T+w} \times \frac{m_R}{m_R+m_T} - (e\times c_R)$ and $u_T(IWc) = \frac{m_R+m_T+w}{m_D+m_R+m_T+w} \times \frac{m_T}{m_R+m_T} - (e\times c_T)$. Then efficiency gains do not change why R and T fight because $\frac{\partial u_R(IWc)}{\partial w} > 0$ and $\frac{\partial u_T(IWc)}{\partial w} > 0$, and $\frac{\partial u_R(IWc)}{\partial m_T} < 0$ and $\frac{\partial u_T(IWc)}{\partial m_R} < 0$.

Furthermore, efficiency gains do not affect when R and T fight, because D's cut-point strategies do not change substantively. Efficiency gains only affect D's chances of winning an interstate war, so D is indifferent between retaliating and tolerating when $u_D(IWc) = u_D(ICWc)$, which simplifies to $c_D^{\oplus} = \frac{\left(\frac{m_D}{m_D+m_R+m_T+w} - \frac{m_D\pi}{m_D+m_R+s}\right)}{e-1}$, or $c_D^{\oplus} = \frac{(p_D^{IWc}-p_D^{ICW}\pi)}{e-1}$, where $p_D^{IWc} > p_D^{IW}$ by assumption and $\frac{\partial c_D^{\oplus}}{\partial w} < 0$. Therefore, efficiency gains strictly makes retaliation less likely, while making R and T more willing to fight.

References

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